

# MATH 0017 - CONCEPTS OF MATHEMATICS

## Catalog Description

Prerequisite: Three years of high school mathematics which includes two years of algebra and one year of geometry; or MATH D and B with grades of "C" or better; or placement by matriculation assessment process

Hours: 54 lecture

Description: Exploration of mathematical patterns and relations, formulation of conjectures based on the explorations, proving (or disproving) the conjectures. Includes different problem solving techniques, number theory, probability, statistics, sequences and series, and geometry. Intended for students interested in elementary education. (CSU, UC-with unit limitation)

## Course Student Learning Outcomes

- CSLO #1: Develop and implement strategies for approaching unfamiliar mathematical problems.
- CSLO #2: Evaluate, improve, and correct orally presented or written solutions.
- CSLO #3: Logically present clear, complete, accurate, and sufficiently detailed solutions to communicate reasoning and demonstrate the method of solving problems.

## Effective Term

Fall 2022

## Course Type

Credit - Degree-applicable

## Contact Hours

54

## Outside of Class Hours

108

## Total Student Learning Hours

162

## Course Objectives

For each topic, the students will:

1. Develop a strategy for approaching problems with which they are unfamiliar;
2. Construct clear and logical solutions or proofs for each problem;
3. Evaluate orally presented or written solutions for flaws and/or mistakes and correct these flaws or mistakes.

### I. PROBLEM SOLVING TECHNIQUES

- A. Examine and organize information in unfamiliar problems as an initial approach to solving;
  - B. construct tables, graphs, and diagrams and utilize as a problem solving technique;
  - C. utilize algebraic solutions to presented problems, which include systems of equations in solution;
- ### II. NUMBER THEORY

A. Propose, test, debate, and construct a clear, logical, and sound solution to these problems in groups;

B. Solve other problems using the Euclidean Algorithm;

### III. SET THEORY

A. Categorize information in a problem into clear sets, subsets, and complementary sets;

B. Calculate the number of elements in intersections and unions of sets using Venn Diagrams;

### IV. PROBABILITY AND STATISTICS

A. Propose, test, debate, and construct a solution to the Buffon Needle (Noodle) Problem based on experimental data;

B. Solve problems using counting techniques, which include the use of combinatorics;

C. Select the best solution to a problem using probability and expected values;

### V. GEOMETRY AND NETWORKS

A. Propose, test, debate, and construct a solution to the Highway Inspector Problem (an adaptation of Eulerian Networks);

B. Design Eulerian and Hamiltonian networks with given numbers of vertices and test them for transportivity;

C. Propose, test, debate, and construct solutions to open-ended problems involving geometry including Sperner's Lemma;

### VI. SEQUENCES AND SERIES

A. Propose, test, debate, and construct a solution to the Handshake Problem and its variations, which includes summation of finite series;

B. Create solutions and verify their validity to infinite sum problems in the form of geometric sequences and series;

C. Predict the entries in a sequence by following the pattern in a sequence;

D. Construct a series that correctly represents information in a problem and find its sum, may be finite or infinite.

## General Education Information

- Approved College Associate Degree GE Applicability
  - AA/AS - Comm & Analyt Thinking
  - AA/AS - Mathematical Skills
- CSU GE Applicability (Recommended-requires CSU approval)
  - CSUGE - B4 Math/Quantitative Reasoning
- Cal-GETC Applicability (Recommended - Requires External Approval)
- IGETC Applicability (Recommended-requires CSU/UC approval)

## Articulation Information

- CSU Transferable
- UC Transferable

## Methods of Evaluation

- Objective Examinations
  - Example: Example: On checking 200 students, it is found that 70 are taking French, 40 are taking German, 75 are taking Spanish, 10 are taking French and German, 30 are taking French and Spanish, 15 are taking German and Spanish, and 70 are taking no language. If it is known that no students are taking all three languages, draw a Venn Diagram to determine the answers to each of the following questions: a) How many are taking two languages? b) How many are taking only Spanish? c) How many are taking Spanish and not French?
- Problem Solving Examinations
  - Example: Example: Construct two networks, each with 5 vertices, such that one of them is traversable exactly once, and the other

is not. Explain your answer. Evaluation: Students will be evaluated using the following criteria: 1) Mathematical correctness of their answer, 2) Mathematical correctness of their solution strategy, and 3) effectiveness at communicating mathematical concepts.

## Repeatable

No

## Methods of Instruction

- Lecture/Discussion
- Distance Learning

Lecture:

1. The instructor poses a problem, such as the handshake problem: "If everyone in this room shook hands once with every other person in this room, how many handshakes occur?" The students are given time to work in groups to come up with an answer to the problem. The instructor monitors group progress, interjects hints or ideas as they work, and then asks students to share their solutions with the class at the end. The instructor also attempts to help students generalize their answer to a mathematical formula (Objective III).
2. Students are required to read materials on the Golden Ratio (text, research documents) before coming to class. The instructor has the students discuss the readings with each other in groups. The instructor then allows time for whole-class question and answer and attempts to highlight the essential concepts from the readings. These readings may then be used by the instructor to launch class activities (Objective V).

Distance Learning

1. The instructor poses a discussion board topic, such as the handshake problem: "If everyone in this room shook hands once with every other person in this room, how many handshakes occur?" The students collaborate in virtual groups to come up with an answer to the problem. The instructor monitors group progress, interjects hints or ideas, and then asks students to post their solutions to the discussion board. The instructor also attempts to help students generalize their answer to a mathematical formula (Objective III).
2. Students are required to read materials on the Golden Ratio (text, research documents) before joining discussion board. The instructor has the students take part in small group discussions on the readings. Then instructor then initiates a whole-class discussion board where students highlight the essential concepts from the readings. These readings may then be used by the instructor to launch class activities (Objective V).

## Typical Out of Class Assignments

### Reading Assignments

1. Find an internet source on Venn Diagrams and come to class prepared to discuss the logic of the Venn Diagram.
2. Read the homework handouts to determine the questions being asked and the work that will need to be done to accomplish the solution.
3. Read a solution to a problem prepared by another group and analyze that solution for correct logic or implied flaws.
4. Read article "Teaching Mathematics Requires Special Skills" by Debbie Ball (or similar article on same topic). Write journal entry and discuss in class.

## Writing, Problem Solving or Performance

1. Working in groups, develop a possible solution for the "Highway Inspector" network problem. Test the conjecture for accuracy and write up a clear, logical proof for the solution.
2. Within a group that has discovered a flaw with another group's solution to a problem, write a paper indicating how the solution was in error and a proposal on how to fix that error.

## Other (Term projects, research papers, portfolios, etc.)

1. Geometry Group Project - Polyhedra building/investigation dualism, stellating, truncating, and compounds.
2. Collection and organization of experimental data for Buffon Needle (Noodle) problem.
3. Research historical math approaches to various problems given in class, with use of a library or internet.
4. Research mathematicians past or present and give presentation in class, with use of library or internet.

## Required Materials

- Mathematical Ideas
  - Author: Charles Miller, Vern Heeren
  - Publisher: Pearson
  - Publication Date: 2020
  - Text Edition: 14th
  - Classic Textbook?: No
  - OER Link:
  - OER:

## Other materials and-or supplies required of students that contribute to the cost of the course.