

# MATH 0033 - DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

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## Catalog Description

Prerequisite: Completion of MATH 31 with grade of "C" or better

Advisory: Completion of MATH 32 with grade of "C" or better strongly recommended

Hours: 108 lecture

Description: First and second order ordinary differential equations, linear differential equations, numerical methods and series solutions, Laplace transforms, modeling and stability theory, systems of linear differential equations, matrices, determinants, vector spaces, linear transformations, orthogonality, eigenvalues and eigenvectors. (C-ID MATH 910S) (CSU, UC)

## Course Student Learning Outcomes

- CSLO #1: Solve first and higher order ordinary and linear differential equations; using Laplace transformations, numerical, and series methods.
- CSLO #2: Utilize theorems from linear algebra and use matrices to solve systems of equations, including differential equations.
- CSLO #3: Utilize theorems from linear algebra to classify sets and mappings.
- CSLO #4: Present clear, complete, accurate, and sufficiently detailed solutions to communicate reasoning and demonstrate the method of solving problems.

## Effective Term

Spring 2021

## Course Type

Credit - Degree-applicable

## Contact Hours

108

## Outside of Class Hours

216

## Total Student Learning Hours

324

## Course Objectives

1. Create and analyze mathematical models using ordinary differential equations;
2. Verify solutions of differential equations;
3. Identify the type of a given differential equation and select and apply the appropriate analytical technique for finding the solution of first order and selected higher order ordinary differential equations;
4. Apply the existence and uniqueness theorems for ordinary differential equations;
5. Find power series solutions to ordinary differential equations including Frobenius solutions;

6. Determine the Laplace Transform and inverse Laplace Transform of functions and use to solve differential equations with initial value conditions;
7. Solve Linear Systems of ordinary differential equations;
8. Find solutions of systems of equations using various methods appropriate to lower division linear algebra;
9. Use bases and orthonormal bases to solve problems in linear algebra;
10. Find the dimension of spaces such as those associated with matrices and linear transformations;
11. Find eigenvalues and eigenvectors and use them in applications;
12. Prove basic results in linear algebra using appropriate proof-writing techniques such as linear independence of vectors; properties of subspaces; linearity, injectivity and surjectivity of functions; and properties of eigenvectors and eigenvalues;
13. Verify that the axioms of a vector space, subspace, and inner product are satisfied or cannot be satisfied for a variety of sets including:  $n$ -dimensional space, polynomials, matrices, continuous and differentiable functions;
14. Examine Legendre and Bessel differential equations and their solutions;
15. Examine the phase plane for generating a qualitative representation of the solution to a system of nonlinear differential equations.

## General Education Information

- Approved College Associate Degree GE Applicability
  - AA/AS - Comm & Analyt Thinking
  - AA/AS - Mathematical Skills
- CSU GE Applicability (Recommended-requires CSU approval)
  - CSUGE - B4 Math/Quantitative Reasoning
- Cal-GETC Applicability (Recommended - Requires External Approval)
- IGETC Applicability (Recommended-requires CSU/UC approval)
  - IGETC - 2A Math/Quan Reasoning

## Articulation Information

- CSU Transferable
- UC Transferable

## Methods of Evaluation

- Objective Examinations
  - Example: Prove that  $P_3$  is a vector space by verifying that the set  $P_3$  satisfies each of the axioms for a vector space. This problem is graded for completeness and accuracy. Students need to verify each of the ten vector space axioms.
- Problem Solving Examinations
  - Example: A tank initially contains 20 L of water. A solution containing 1 g/L of chemical flows into the tank at a rate of 3 L/min, and the mixture flows out at a rate of 2 L/min. Set up the initial-value problem for  $A(t)$  and then solve the initial-value problem for  $A(t)$ . This problem is graded for completeness and accuracy of two parts: 1. The set-up of the initial-value problem and 2. The solving of the initial-value problem.
- Projects
  - Example: Read from the text and research a mathematical structure similar to a vector space called a lattice. In a report, give specific examples of 5 different types of lattices and prove the corresponding bases for each of your examples. In a summary, compare your proofs performed with lattices with the proofs for bases you performed with vector spaces and explain how they are similar and different. This report is graded on completeness, and

accuracy of the proofs. The student will also be graded on their summary and their analytical comparison of the proofs in the two mathematical structures.

- Reports
  - Example: Read from the text and research a mathematical structure similar to a vector space called a lattice. In a report, give specific examples of 5 different types of lattices and prove the corresponding bases for each of your examples. In a summary, compare your proofs performed with lattices with the proofs for bases you performed with vector spaces and explain how they are similar and different. This report is graded on completeness, and accuracy of the proofs. The student will also be graded on their summary and their analytical comparison of the proofs in the two mathematical structures.

## Repeatable

No

## Methods of Instruction

- Lecture/Discussion
- Distance Learning

Lecture:

1. The students and instructor will engage in an interactive discussion concerning whether  $P_3$  with certain restrictions constitutes a vector space. The discussion will lead to a conclusion that prompts the instructor to introduce the writing technique necessary for students to verify that such a set is a vector space or introduce the writing technique necessary for students to verify such a set is not a vector space. Students will then get an opportunity to practice such writing techniques on other sets provided by the instructor. (Objective 3)
2. The instructor will direct students to form small teams of 2 to 4 to perform at least two of the listed methods to find solutions to a system of equations. Instructors will list the variety of methods on the board. Instructor will facilitate and direct individual groups to complete specific methods and have students share their work at the white board with the rest of the class. (Objective 8)

Distance Learning

1. The students and instructor will engage in a discussion board concerning whether  $P_3$  with certain restrictions constitutes a vector space. The discussion will lead to a conclusion that prompts the instructor to introduce the writing technique necessary for students to verify that such a set is a vector space or introduce the writing technique necessary for students to verify such a set is not a vector space. Students will then get an opportunity to practice such writing techniques on other sets provided by the instructor. (Objective 3)
2. The instructor will form small virtual groups of 2 to 4 to post at least two of the listed methods to find solutions to a system of equations. Instructors will list the variety of methods on the board. Instructor will facilitate and direct individual groups to complete specific methods and have students post their work for peer review. (Objective 8)

## Typical Out of Class Assignments

### Reading Assignments

1. Read in the textbook about the axioms of a vector space. Come to class prepared to discuss what it means to be a vector space and what it means not to be a vector space.
2. Read in your textbook (and research

online) slope fields of the form  $D(y)=f(x,y)$  including isoclines, equilibrium solutions, and concavity.

3. Read in the textbook about the projection vector and examine how it is used to derive the formula in the Gram-Schmidt Process.

## Writing, Problem Solving or Performance

1. Sketch the slope field and some representative solution curves for the differential equation  $D(y)=y(y-1)$ .
2. Use technology to graph the slope field and validate the algebraic calculations of isoclines, equilibrium solutions, and concavity.

## Other (Term projects, research papers, portfolios, etc.)

### Required Materials

- Differential Equations and Linear Algebra
  - Author: Goode & Annin
  - Publisher: Pearson
  - Publication Date: 2017
  - Text Edition: 4th
  - Classic Textbook?: No
  - OER Link:
  - OER:
- Differential Equations and Linear Algebra
  - Author: Edwards & Penney & Calvis
  - Publisher: Pearson
  - Publication Date: 2017
  - Text Edition: 4th
  - Classic Textbook?: No
  - OER Link:
  - OER:

## Other materials and-or supplies required of students that contribute to the cost of the course.